

Surface Areas and Volumes

Selected NCERT Questions

1. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. Here, radius of hemisphere = radius of cylinder = r cm
 $= 7$ cm

and height of cylinder, $h = (13 - 7)$ cm = 6 cm

Now, inner surface area of the vessel

$$\begin{aligned} &= \text{curved surface area of the cylindrical part} + \text{curved surface area of hemispherical part} \\ &= (2\pi rh + 2\pi r^2) = 2\pi r (h + r) \\ &= 2 \times \frac{22}{7} \times 7 \times (6 + 7) \\ &= 2 \times 22 \times 13 = 572 \text{ cm}^2 \end{aligned}$$

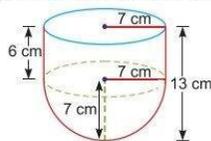


Fig. 12.5

2. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

OR

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol. We have, $CD = 15.5$ cm and $OB = OD = 3.5$ cm

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, $r = OB = 3.5$ cm

$$h = OC = CD - OD = (15.5 - 3.5) \text{ cm} = 12 \text{ cm}$$

$$l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ cm}$$

Also, radius of the hemisphere, $r = 3.5$ cm

\therefore Total surface area of the toy

$$\begin{aligned} &= \text{surface area of cone} + \text{surface area of hemisphere} \\ &= \pi rl + 2\pi r^2 = \pi r(l + 2r) \\ &= \frac{22}{7} \times 3.5 (12.5 + 2 \times 3.5) \\ &= \frac{22}{7} \times 3.5 \times 19.5 = 214.5 \text{ cm}^2 \end{aligned}$$

[CBSE (AI) 2017]

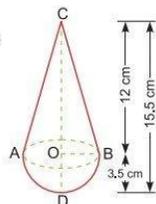


Fig. 12.6

3. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. [Competency Based Question]

Sol. The greatest diameter that a hemisphere can have = 7 cm = l

Radius of the hemisphere (R) = $\frac{7}{2}$ cm

\therefore Surface area of the solid after surmounting hemisphere

$$\begin{aligned} &= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2 \\ &= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \end{aligned}$$

$$\begin{aligned}
 &= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \\
 &= 294 + 38.5 = 332.5 \text{ cm}^2
 \end{aligned}$$

4. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (Fig. 12.7). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

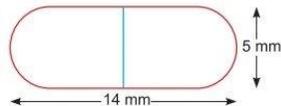


Fig. 12.7

- Sol. Let the radius and height of the cylinder be r cm and h cm respectively. Then,

$$r = \frac{5}{2} \text{ mm} = 2.5 \text{ mm}$$

$$\text{and } h = (14 - 2 \times \frac{5}{2}) \text{ mm} = 9 \text{ mm}$$

$$\text{Also, radius of hemisphere } r = \frac{5}{2} \text{ mm}$$

Now, surface area of the capsule

$$= \text{Curved surface area of cylinder} + \text{Surface area of two hemispheres}$$

$$= 2\pi rh + 2 \times 2\pi r^2 = 2\pi r (h + 2r)$$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times (9 + 2 \times \frac{5}{2})$$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 14 = 220 \text{ mm}^2$$

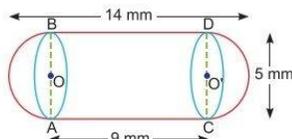


Fig. 12.8

5. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2 . (Note that the base of the tent will not be covered with canvas).

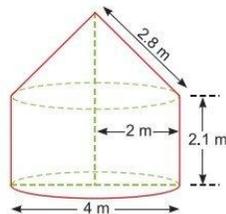


Fig. 12.9

- Sol. We have,

$$\text{Radius of cylindrical base} = \frac{4}{2} = 2 \text{ m}$$

$$\text{Height of cylindrical portion} = 2.1 \text{ m}$$

$$\therefore \text{Curved surface area of cylindrical portion} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 = 26.4 \text{ m}^2$$

$$\text{Radius of conical base} = 2 \text{ m}$$

$$\text{Slant height of conical portion} = 2.8 \text{ m}$$

$$\therefore \text{Curved surface area of conical portion} = \pi rl$$

$$= \frac{22}{7} \times 2 \times 2.8 = 17.6 \text{ m}^2$$

$$\text{Now, total area of the canvas} = (26.4 + 17.6) \text{ m}^2 = 44 \text{ m}^2$$

$$\therefore \text{Total cost of the canvas used} = ₹ 500 \times 44$$

$$= ₹ 22,000$$

6. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 .

Sol. We have,

$$\text{Radius of the cylinder} = \frac{1.4}{2} = 0.7 \text{ cm}$$

$$\text{Height of the cylinder} = 2.4 \text{ cm}$$

$$\text{Also, radius of the cone} = 0.7 \text{ cm}$$

$$\text{and height of the cone} = 2.4 \text{ cm}$$

$$\text{Now, slant height of the cone} = \sqrt{(0.7)^2 + (2.4)^2}$$

$$\Rightarrow l = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5 \text{ cm}$$

$$\therefore \text{Total surface area of the remaining solid}$$

$$= \text{curved surface area of cylinder} + \text{curved surface area of the cone}$$

$$+ \text{area of upper circular base of cylinder}$$

$$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 0.7 \times [2 \times 2.4 + 2.5 + 0.7] = 22 \times 0.1 \times (4.8 + 2.5 + 0.7)$$

$$= 2.2 \times 8.0 = 17.6 \text{ cm}^2 \approx 18 \text{ cm}^2$$

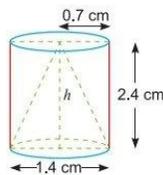


Fig. 12.10

7. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Sol. We have,

Height of cone is equal to its radius

$$\text{i.e., } h = r = 1 \text{ cm} \quad (\text{Given})$$

Also, radius of hemisphere = $r = 1 \text{ cm}$

Now, volume of the solid

$$= \text{volume of the cone} + \text{volume of the hemisphere}$$

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$= \frac{1}{3} \pi r^2 \times r + \frac{2}{3} \pi r^3 \quad [\because h = r]$$

$$= \frac{\pi r^3}{3} + \frac{2}{3} \pi r^3 = \pi r^3 = \pi(1)^3 = \pi \text{ cm}^3$$

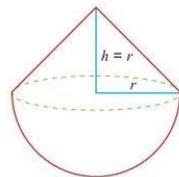


Fig. 12.11

8. A gulab jamun, contains sugar syrup about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (Fig. 12.12).

Sol. We have,

Radius of cylindrical portion and hemispherical portion of a gulab jamun

$$= \frac{2.8}{2} = 1.4 \text{ cm}$$

$$\text{Length of cylindrical portion} = 5 - 1.4 - 1.4 = 2.2 \text{ cm}$$

Volume of one gulab jamun

$$= \text{volume of the cylindrical part} + \text{volume of the hemispherical ends}$$

$$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \pi r^2 h + \frac{4}{3} \pi r^3$$

$$= \pi r^2 \left(h + \frac{4}{3} r \right) = \frac{22}{7} \times (1.4)^2 \left(2.2 + \frac{4}{3} \times 1.4 \right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(\frac{6.6 + 5.6}{3} \right) = \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

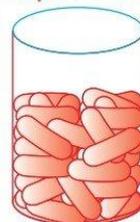


Fig. 12.12

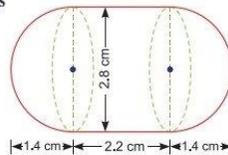


Fig. 12.13

$$\therefore \text{Volume of 45 gulab jamuns} = 45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

$$\begin{aligned} \therefore \text{Quantity of syrup in 45 gulab jamuns} &= 30\% \text{ of their volume} \\ &= \frac{30}{100} \times 45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3} \\ &= 338.184 \text{ cm}^3 \\ &= 338 \text{ cm}^3 \text{ (approx.)} \end{aligned}$$

9. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (Fig. 12.14).

Sol. We have,

Length of cuboid = $l = 15$ cm

Breadth of cuboid = $b = 10$ cm

Height of cuboid = $h = 3.5$ cm

And radius of conical depression = 0.5 cm

Depth of conical depression = 1.4 cm

Now, volume of wood in the entire pen stand

$$= \text{volume of cuboid} - 4 \times \text{volume of a conical depression}$$

$$= lbh - 4 \times \frac{1}{3} \pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$$

$$= (525 - 1.47) \text{ cm}^3 = 523.53 \text{ cm}^3$$

10. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. (Use $\pi = 3.14$).

Sol. Let r_1 and h_1 be the radius and height of longer cylinder, respectively, and r_2, h_2 be the respective radius and height of smaller cylinder mounted on the longer cylinder. Then we have,

$$r_1 = 12 \text{ cm}, \quad h_1 = 220 \text{ cm}$$

$$r_2 = 8 \text{ cm}, \quad h_2 = 60 \text{ cm}$$

Now, volume of solid iron pole

$$= \text{Volume of the longer cylinder} + \text{Volume of smaller cylinder}$$

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

$$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$

$$= 3.14 \times 144 \times 220 + 3.14 \times 64 \times 60$$

$$= 99475.2 + 12057.6$$

$$= 111532.8 \text{ cm}^3$$

Hence, the mass of the pole = (111532.8×8) grams

$$= \frac{111532.8 \times 8}{1000} \text{ kg}$$

$$= 892.2624 \text{ kg}$$

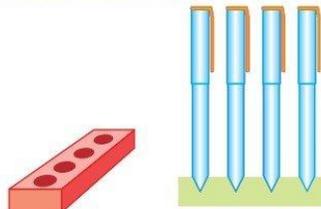


Fig. 12.14

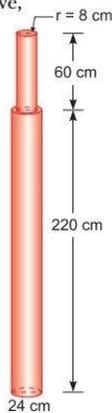


Fig. 12.15

11. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3 . Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Sol. We have,

Radius of cylindrical neck = 1 cm and height of cylindrical neck = 8 cm

Radius of spherical part = 4.25 cm

$$\begin{aligned} \text{Now, volume of spherical vessel} &= \pi r^2 h + \frac{4}{3} \pi r^3 \\ &= \pi(1^2) \times 8 + \frac{4}{3} \times \pi \times (4.25)^3 \\ &= 3.14 \times \left[8 + \frac{4}{3} \times (4.25)^3 \right] \\ &= 3.14 \times [8 + 102.354] \\ &= 3.14 \times 110.354 \\ &= 346.51 \text{ cm}^3 \end{aligned}$$

\therefore The answer found by the child is incorrect.

Hence, the correct answer is 346.51 cm^3 .

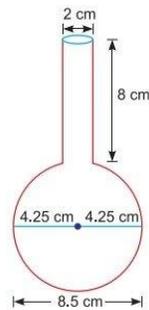


Fig. 12.16

12. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Sol. Volume of the cylinder = $\pi R^2 H$

$$\begin{aligned} &= \frac{22}{7} \times (6)^2 \times 15 \\ &= \frac{22}{7} \times 36 \times 15 \text{ cm}^3 \end{aligned}$$

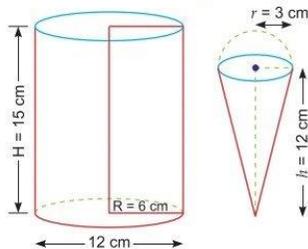


Fig. 12.17

$$\begin{aligned} \text{Now, volume of cone having hemispherical shape on top} &= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= \frac{1}{3} \pi r^2 (h + 2r) \\ &= \frac{1}{3} \times \frac{22}{7} \times (3)^2 (12 + 2 \times 3) \\ &= \frac{1}{3} \times \frac{22}{7} \times 9 \times 18 = \frac{22}{7} \times 54 \text{ cm}^3 \end{aligned}$$

\therefore The number of cones that can be filled with ice cream

$$\begin{aligned} &= \frac{\text{Volume of cylinder}}{\text{Volume of cone having hemispherical shape on top}} \\ &= \frac{\frac{22}{7} \times 36 \times 15}{\frac{22}{7} \times 54} = 10 \end{aligned}$$

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. A medicine-capsule is in the shape of a cylinder of radius 0.25 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. What is the total surface area of the capsule? (Take π as 3.14) [Competency Based Question]



Fig. 12.18

- (a) 0.785 cm^2 (b) 0.98125 cm^2 (c) 2.7475 cm^2 (d) 3.14 cm^2
2. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?
 (a) $2k^3$ (b) $3k^3$ (c) $4k^3$ (d) $6k^3$
3. The radius of a sphere (in cm) whose volume is $12\pi \text{ cm}^3$, is [CBSE 2020(30/5/1)]
 (a) 3 cm (b) $3\sqrt{3}$ cm (c) $3^{2/3}$ cm (d) $3^{1/3}$ cm
4. A mason constructs a wall of dimensions $270 \text{ cm} \times 300 \text{ cm} \times 350 \text{ cm}$ with the bricks each of size $22.5 \text{ cm} \times 11.25 \text{ cm} \times 8.75 \text{ cm}$ and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is [NCERT Exemplar]
 (a) 11100 (b) 11200 (c) 11000 (d) 11300
5. A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice cream cones of radius 2 cm and height 7 cm upto its brim. How many children will get the ice cream cones? [NCERT Exemplar]
 (a) 163 (b) 263 (c) 363 (d) 463
6. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is [NCERT Exemplar]
 (a) 9.7 cm^3 (b) 77.6 cm^3 (c) 58.2 cm^3 (d) 19.4 cm^3
7. A solid spherical ball fits exactly inside the cubical box of side $2a$. The volume of the ball is [CBSE 2020(30/C/1)]
 (a) $\frac{16}{3}\pi a^3$ (b) $\frac{1}{6}\pi a^3$ (c) $\frac{32}{3}\pi a^3$ (d) $\frac{4}{3}\pi a^3$
8. A cube of side length 66 cm is filled with spherical metallic balls of diameter 0.6 cm and it is assumed that $\frac{5}{8}$ space of the cube remains unfilled. What are number of balls that can be filled in the cube? (Use $\pi = \frac{22}{7}$) [Competency Based Question]
 (a) 52875 (b) 952875 (c) 1165812 (d) 2264031
9. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is [NCERT Exemplar]
 (a) 142296 (b) 142396 (c) 142496 (d) 142596

10. Abhinav made a model for his school project in the shape of a cylinder of radius 7 cm and height 21 cm, with hemisphere surmounted on one end. He wants to cover the entire model with decorative paper. What is the area, in centimeter square, of paper that is required to cover the model? (Use $\pi = \frac{22}{7}$)

[Competency Based Question]

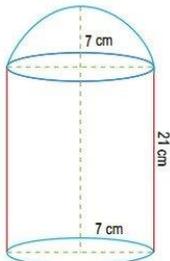


Fig. 12.19

- (a) 976 cm² (b) 1012 cm² (c) 1232 cm² (d) 1386 cm²

Answers

1. (d) 2. (a) 3. (c) 4. (b) 5. (c) 6. (d) 7. (d)
8. (b) 9. (a) 10. (c)

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. If the areas of three adjacent faces of a cuboid are x , y and z respectively, then find the volume of cuboid.

Sol. Let the length, breadth and height of the cuboid are l , b and h respectively.

$$\therefore x = l \times b; \quad y = b \times h \quad \text{and} \quad z = l \times h$$

$$\Rightarrow xyz = (l \times b) \times (b \times h) \times (l \times h) \\ = l^2 b^2 h^2$$

Now, volume of cuboid = $l \times b \times h$

$$= \sqrt{xyz} \quad (\because l^2 b^2 h^2 = xyz)$$

2. Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

[CBSE 2020(30/3/1)]

Sol. Let height of the cones are h and $3h$ respectively and their radii are $3r$ and r respectively.

$$\therefore \text{Ratio of volume} = \frac{\text{Volume of first cone}}{\text{Volume of second cone}} \\ = \frac{\frac{1}{3}\pi \times (3r)^2 \times h}{\frac{1}{3}\pi \times r^2 \times 3h} = \frac{9}{3} = \frac{3}{1}$$

\therefore Required ratio is 3 : 1.

3. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

[CBSE Delhi 2017]

Sol. As per question

Volume of hemisphere = Surface area of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2} \text{ units}$$

$$\therefore d = 9 \text{ units}$$

4. A solid ball is exactly fitted inside the cubical box of side a . What is the volume of the ball?

Sol. Diameter of the solid ball = edge of the cube = a

$$\therefore \text{Volume of the ball} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3} \times \frac{1}{8}\pi a^3 = \frac{1}{6}\pi a^3$$

Short Answer Questions-I

Each of the following questions are of 2 marks.

1. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7} \text{ cm}^3$. Find the height of the cylinder. [Use $\pi = \frac{22}{7}$] [CBSE 2020(30/2/1)]

Sol. We have, in a right circular cylinder

$$\text{Height} = \text{Radius} \Rightarrow h = r$$

$$\text{and volume of cylinder} = 25\frac{1}{7} \text{ cm}^3$$

$$\Rightarrow \pi r^2 h = \frac{176}{7}$$

$$\Rightarrow \pi \times h^2 \times h = \frac{176}{7} \quad (\because h = r)$$

$$\Rightarrow \frac{22}{7} \times h^3 = \frac{176}{7}$$

$$\Rightarrow h^3 = \frac{176}{22} = 8 \Rightarrow h^3 = (2)^3 \Rightarrow h = 2 \text{ cm}$$

\therefore Height of the cylinder = 2 cm

2. In Fig. 12.20, a tent is in the shape of a cylinder surmounted by a conical top. The cylindrical part is 3.5 m high and conical part has slant height 4.2 m. Both the parts have same radius 2 m. Find the area of the canvas used to make the tent. (Use $\pi = \frac{22}{7}$)

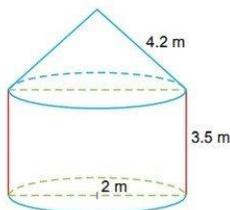


Fig. 12.20

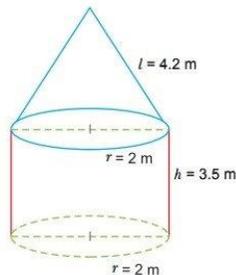


Fig. 12.21

Sol. We have,

Area of the canvas used to make the tent

$$= \text{CSA of cylinder} + \text{CSA of conical part}$$

$$= 2\pi rh + \pi rl$$

$$= 2 \times \frac{22}{7} \times 2 \times 3.5 + \frac{22}{7} \times 2 \times 4.2$$

$$= 44 + 26.4 = 70.4 \text{ m}^2$$

3. If the total surface area of a solid hemisphere is 462 cm^2 , find its volume. [Take $\pi = \frac{22}{7}$]
[CBSE (AI) 2015]

Sol. Given, total surface area of solid hemisphere = 462 cm^2

$$\Rightarrow 3\pi r^2 = 462 \text{ cm}^2$$

$$3 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = 49 \quad \Rightarrow r = 7 \text{ cm}$$

$$\text{Volume of solid hemisphere} = \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.67 \text{ cm}^3$$

4. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.
[CBSE (AI) 2015]

Sol. Radius of hemispherical bowl, $R = \frac{36}{2} = 18 \text{ cm}$

Radius of cylindrical bottle, $r = 3 \text{ cm}$

Let height of cylindrical bottle = h

Since, 10% liquid is wasted, therefore only 90% liquid is filled into 72 cylindrical bottles.

\therefore Volume of 72 cylindrical bottles = 90% of the volume in bowl

$$\Rightarrow 72 \times \pi r^2 h = 90\% \text{ of } \frac{2}{3} \pi R^3$$

$$72 \times \pi \times 3 \times 3 \times h = \frac{90}{100} \times \frac{2}{3} \times \pi \times 18 \times 18 \times 18$$

$$h = \frac{90 \times 2 \times \pi \times 18 \times 18 \times 18}{100 \times 3 \times \pi \times 72 \times 3 \times 3}$$

$$h = \frac{27}{5} = 5.4 \text{ cm}$$

5. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid.
(Use $\pi = \frac{22}{7}$)
[CBSE 2019(30/2/1)]

Sol. Height of cylinder = $20 - 7 = 13 \text{ cm}$.

$$\text{Total volume} = \pi \left(\frac{7}{2}\right)^2 \cdot 13 + \frac{4}{3} \pi \left(\frac{7}{2}\right)^3 \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{49}{4} \left(13 + \frac{4}{3} \cdot \frac{7}{2}\right) \text{ cm}^3$$

$$= \frac{77 \times 53}{6} = 680.17 \text{ cm}^3$$

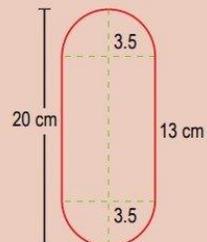


Fig. 12.22

Short Answer Questions-II

Each of the following questions are of 3 marks.

1. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

[CBSE (AI) 2017(30/3)]

Sol.

speed of water in canal = 25 km/hr.
in 40 min = $\frac{40}{60} = \frac{2}{3}$ hr,
length of water = $25 \times \frac{2}{3} = \frac{50}{3}$ km = $\frac{50000}{3}$ m
volume of water in canal in 40 minutes = volume of water for irrigation.
 $\frac{5.4}{10} \times \frac{1.8}{10} \times \frac{50000}{3} \text{ m}^3 = \frac{10}{100} \times l \times b \times m^3$
 $324 \times 5000 = l \times b$
~~1600~~ $1620000 = l \times b$
area irrigated in 40 minutes is
 1620000 m^2
 $= \frac{1620000}{1000000}$
 $= 1.62 \text{ km}^2$ or 162 hectares.

[Topper's Answer 2017]

2. 500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?

[CBSE 2020(30/3/1)]

Sol. Let h m be the rise of water level in the pond.

$$\therefore \text{Displacement of water by 500 persons} = l \times b \times h$$

$$\Rightarrow 500 \times 0.04 = 80 \times 50 \times h$$

$$\Rightarrow 20 = 4000h$$

$$\therefore h = \frac{20}{4000} = \frac{1}{200} \text{ m} = \frac{100}{200} \text{ cm} = \frac{1}{2} \text{ cm} = 0.5 \text{ cm}$$

3. A sphere of diameter 6 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by 4 cm. Find the diameter of the cylindrical vessel.

[CBSE (AI) 2016]

Sol. Volume of sphere = $\frac{4}{3} \pi (3)^3 \text{ cm}^3$

$$\text{Volume of water rise in cylinder} = \pi r^2 h = \pi r^2 \cdot 4 \quad [\because h = 4 \text{ cm}]$$

$$\therefore \pi r^2 \cdot 4 = \frac{4}{3} \pi (3)^3$$

$$\Rightarrow r^2 = \frac{27}{3} = 9$$

$$\Rightarrow r = 3 \text{ cm}$$

4. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? [CBSE 2018(30/3)]

Sol.

2) Conical heap of rice:
 (choice 2) Dimensions: diameter = 24m, height 3.5 m \rightarrow radius = 12m.

Volume of cone = $\frac{1}{3} \times \pi r^2 h$ cu. units.
 $= \frac{1}{3} \times \frac{22}{7} \times 12^2 \times 3.5$ cu. m.
 $= 132 \times 4$
 $= 528$ cu. m.

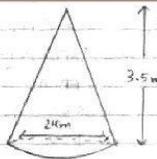
The volume of the rice heap is 528 cu. m.

Area of cloth required = Curved surface area.
 CSA of cone = $\pi r l$ sq. units. where $l = \sqrt{h^2 + r^2}$ units.
 Finding l : $l = \sqrt{3.5^2 + 12^2}$ units
 $= \sqrt{12.25 + 144}$
 $= \sqrt{156.25}$
 $= 12.5$ m.

\Rightarrow CSA = $\pi r l$ sq. units
 $= \frac{22}{7} \times 12.5 \times 12$
 $= \frac{22 \times 150}{7} = \frac{3300}{7} = 471.428571 \text{ m}^2$.

The area of canvas cloth required is 471.428571 m².

[Topper's Answer 2018]



5. The $\frac{3}{4}$ th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel. [CBSE Delhi 2017]

Sol. Let the height of cylindrical vessel be h cm.

According to question,

$$\frac{3}{4} \times \text{volume of conical vessel} = \text{volume of cylindrical vessel}$$

$$\Rightarrow \frac{3}{4} \times \frac{1}{3} \times \pi \times 5 \times 5 \times 24 = \pi \times 10 \times 10 \times h$$

$$\Rightarrow h = \frac{3}{2} \text{ cm or } 1.5 \text{ cm}$$

6. Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (Fig. 12.23). The height of the cylinder is 1.45 m and its radius is 30 cm.

Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$) [Competency Based Question]

Sol. Let h be height of the cylinder, and r be the common radius of the cylinder and hemisphere.

Then, the total surface area of the bird-bath

$$= \text{curved surface area of cylinder} + \text{curved surface area of hemisphere}$$

$$= 2\pi r h + 2\pi r^2 = 2\pi r (h + r)$$

$$= 2 \times \frac{22}{7} \times 30 (145 + 30) \text{ cm}^2$$

$$= 33,000 \text{ cm}^2 = 3.3 \text{ m}^2$$

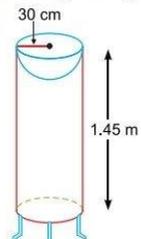


Fig. 12.23

7. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. [Use $\pi = \frac{22}{7}$] [CBSE (AI) 2016(30/2)]

Sol.

12) Radius & height of conical vessel = 5 cm & 24 cm resp.
 Volume of cone = $\frac{1}{3}\pi r^2 h$
 Volume of cone = $\frac{1}{3}\pi \times 25 \times 24 \text{ cm}^3$
 Water is emptied of cylindrical vessel of $r = 10 \text{ cm}$ & height = h
 Volume of cone = Volume of cylinder
 $\Rightarrow \frac{1}{3}\pi \times 25 \times 24 = \pi \times 10 \times 10 \times h$
 $= \frac{200}{100} \text{ cm} = h$
 $= \boxed{2 \text{ cm} = h}$

$\left. \begin{array}{l} \text{Volume of -} \\ \text{Cone} = \frac{1}{3}\pi r^2 h \\ \text{Cylinder} = \pi r^2 h \end{array} \right\}$

[Topper's Answer 2016]

8. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly, filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel. [CBSE (AI) 2016 (30/2)]

Sol.

14) Diameter of sphere = 12 cm
 Its radius = 6 cm
 Volume = $\frac{4}{3}\pi \times 6^3 \text{ cm}^3$ [Volume of sphere = $\frac{4}{3}\pi r^3$]
 It is submerged into water, in cylindrical vessel, then water level rise by $3\frac{5}{9} \text{ cm} = \frac{32}{9} \text{ cm}$
 Volume submerged = Volume rise
 Let radius of cylinder be $r \text{ cm}$
 $\Rightarrow \frac{4}{3}\pi \times 6^3 = \pi \times r^2 \times \frac{32}{9}$
 $27 \times 216 \times 3 \times 4 = r^2$
 324
 $\Rightarrow 4 \times 27 \times 3 = \frac{r^2}{4} \Rightarrow 4 \times 81 \text{ cm}^2 = r^2$
 $r = 9 \text{ cm}$
 Diameter = $2r = 9 \text{ cm} \times 2 = \boxed{18 \text{ cm}}$

[Topper's Answer 2016]

9. The radius and height of a solid right circular cone are in the ratio of 5 : 12. If its volume is 314 cm^3 , find its total surface area. [Take $\pi = 3.14$] [CBSE (F) 2017]

Sol. Given $r : h = 5 : 12$

$$\text{Let } r = 5x \text{ and } h = 12x$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$314 = \frac{1}{3} \times 3.14 (5x)^2 \times 12x$$

$$\Rightarrow x^3 = \frac{314 \times 3}{3.14 \times 25 \times 12} \Rightarrow x^3 = 1 \Rightarrow x = 1$$

So, the value of $r = 5 \text{ cm}$ and $h = 12 \text{ cm}$

$$\text{Now, } l = \sqrt{(12)^2 + (5)^2} = 13 \text{ cm}$$

$$\begin{aligned} \text{TSA of cone} &= \pi r(l + r) \\ &= 3.14 \times 5 (13 + 5) \\ &= 3.14 \times 90 = 282.6 \text{ cm}^2 \end{aligned}$$

Long Answer Questions

Each of the following questions are of 5 marks.

1. A right cylindrical container of radius 6 cm and height 15 cm is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone. [CBSE 2019(30/2/3)]

Sol. Volume of ice-cream in the cylinder $= \pi(6)^2 \cdot 15 \text{ cm}^3$

$$\begin{aligned} \text{Volume of ice-cream in one cone} &= \frac{1}{3}\pi r^2 \cdot 4r + \frac{2}{3}\pi r^3 \text{ cm}^3 \quad (\text{Given } h = 4r) \\ &= 2\pi r^3 \text{ cm}^3 \end{aligned}$$

$$\Rightarrow 10(2\pi r^3) = \pi(6)^2 \times 15$$

$$\Rightarrow r^3 = (3)^3 \Rightarrow r = 3 \text{ cm.}$$

2. In Fig. 12.24, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm.

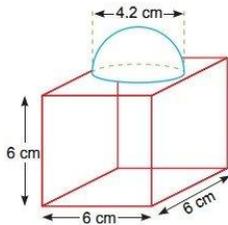


Fig. 12.24

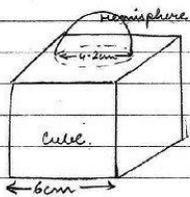
- (i) the total surface area of the block.
 (ii) the volume of the block formed. (Use $\pi = \frac{22}{7}$)

[CBSE 2019(30/3/1)]

Sol.

23. Radius of the hemisphere = $\frac{d}{2}$
 $= \frac{4.2 \text{ cm}}{2}$
 $= 2.1 \text{ cm}$

Side of cube = 6 cm. (a)



a) Total surface area of block = Total surface area of cube + Curved surface area of hemisphere - area enclosed by base of hemisphere

$$= 6a^2 + \frac{4\pi r^2}{2} - \pi r^2$$

$$= 6a^2 + \pi r^2$$

$$= 6 \times 6^2 + \frac{22 \times 7}{7} \times (2.1)^2 \text{ cm}^2$$

$$= 216 + \frac{22 \times 21^2 \times 2.1}{7} \text{ cm}^2$$

$$= (216 + 13.86) \text{ cm}^2$$

$$= 229.86 \text{ cm}^2$$

b) Volume of block formed = Volume of cube + Volume of hemisphere

$$= a^3 + \frac{2}{3}\pi r^3$$

$$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times 2.1^3 \text{ cm}^3$$

$$= 216 + \frac{2}{3} \times \frac{22 \times 21 \times 21 \times 2.1}{1000} \text{ cm}^3$$

$$= 216 + 19.404 \text{ cm}^3$$

$$= 235.404 \text{ cm}^3.$$

[Topper's Answer 2019(30/3/1)]

3. A right triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of double cone thus generated. (Use $\pi = 3.14$). [Competency Based Question]

Sol. In the given Fig. 12.25, ΔPQR is a right triangle, where $PQ = 3$ cm, $PR = 4$ cm and $QR = 5$ cm.

Let $OQ = x \Rightarrow OR = 5 - x$ and $OP = y$

Now in right angled-triangle POQ , we have

$$PQ^2 = OQ^2 + OP^2 \quad (\text{By Pythagoras Theorem})$$

$$\Rightarrow (3)^2 = x^2 + y^2 \Rightarrow y^2 = 9 - x^2 \quad \dots (i)$$

Also from right angled triangle POR , we have

$$OP^2 + OR^2 = PR^2$$

$$\Rightarrow y^2 + (5 - x)^2 = (4)^2$$

$$\Rightarrow y^2 = 16 - (5 - x)^2 \quad \dots (ii)$$

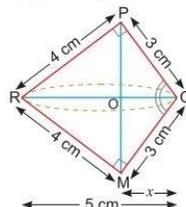


Fig. 12.25

From (i) and (ii), we get

$$9 - x^2 = 16 - (5 - x)^2$$

$$\Rightarrow 9 - x^2 = 16 - (25 + x^2 - 10x)$$

$$\text{or } 9 - x^2 = -9 - x^2 + 10x \quad \Rightarrow \quad 10x = 18 \quad \Rightarrow \quad x = \frac{9}{5}$$

$$\therefore OR = 5 - x = 5 - \frac{9}{5} = \frac{16}{5}$$

Now putting $x = \frac{9}{5}$ in (i), we get

$$y^2 = 9 - \left(\frac{9}{5}\right)^2 = 9 - \frac{81}{25} = \frac{144}{25} \quad \Rightarrow \quad y = \frac{12}{5}$$

$$\therefore OP = y = \frac{12}{5}$$

Now for the cone PQM , radius $OP = \frac{12}{5}$ cm,

$$\text{height } OQ = \frac{9}{5} \text{ cm}$$

$$\therefore \text{Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{12}{5}\right)^2 \times \frac{9}{5} = \frac{432\pi}{125} \text{ cm}^3$$

Also for the cone PRM , radius $OP = \frac{12}{5}$ cm,

$$\text{height } OR = \frac{16}{5} \text{ cm}$$

$$\therefore \text{Volume} = \frac{1}{3}\pi \left(\frac{12}{5}\right)^2 \times \frac{16}{5} = \frac{768\pi}{125} \text{ cm}^3$$

Hence total volume, i.e., volume of the double cone

$$\begin{aligned} &= \left(\frac{432\pi}{125} + \frac{768\pi}{125}\right) = \frac{1200\pi}{125} \\ &= 9.6 \times 3.14 \text{ cm}^3 = 30.144 \text{ cm}^3 \end{aligned}$$

4. Water is flowing through a cylindrical pipe of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm at the rate of 0.7 m/sec. By how much will the water rise in the tank in half an hour? [CBSE Sample Paper 2021]

Sol. For pipe, $r = 1$ cm

Length of water flowing in 1 sec, $h = 0.7$ m = 7 cm

Cylindrical Tank, $R = 40$ cm, rise in water level = H

Volume of water flowing in 1 sec = $\pi r^2 h = \pi \times 1 \times 1 \times 70$

$$= 70\pi$$

Volume of water flowing in 60 sec = $70\pi \times 60$

Volume of water flowing in 30 minutes = $70\pi \times 60 \times 30$

Volume of water in Tank = $\pi R^2 H = \pi \times 40 \times 40 \times H$

Volume of water in Tank = Volume of water flowing in 30 minutes

$$\pi \times 40 \times 40 \times H = 70\pi \times 60 \times 30$$

$$H = 78.75 \text{ cm}$$

[Marking Scheme CBSE Sample Paper 2021]

5. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment. [CBSE 2019(C)(30/1/3)]

Sol. Volume of earth dug out = $\pi \times \frac{9}{4} \times 14 \text{ m}^3$

Radii of circular ring formed = $4 + \frac{3}{2}, \frac{3}{2} \text{ m} = \frac{11}{2} \text{ m}, \frac{3}{2} \text{ m}$

Let height of embankment = h

$\therefore \pi h \left(\frac{121}{4} - \frac{9}{4} \right) = \pi \times \frac{9}{4} \times 14$

$\Rightarrow h = \frac{9}{8} \text{ m or } 1 \frac{1}{8} \text{ m}$

[CBSE Marking Scheme 2019(C)(30/1/3)]

Case Study-based Questions

Each of the following questions are of 4 marks.

- Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551 m^2 . Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1 m^2 , the students put the tents. The radius of the tent is 7 m.

Based on above information answer the following questions.



Fig. 12.26



Fig. 12.27

- What is the volume of cylindrical cup?
- What is the volume of hemispherical cup?

Sol. (i) We have, radius (r) of cylindrical cup = $\frac{7}{2}$ cm

and its height (h) = 10.5 cm

$$\begin{aligned} \therefore \text{Volume of cylindrical cup} &= \pi r^2 h \\ &= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10.5 \\ &= 404.25 \text{ cm}^3 \end{aligned}$$

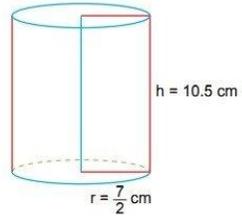


Fig. 12.28

(ii) We have, radius (r) of hemispherical cup = $\frac{7}{2}$ cm

$$\begin{aligned} \therefore \text{Volume of hemispherical cup} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times \frac{7 \times 7 \times 7}{2 \times 2 \times 2} = 89.83 \text{ cm}^3 \end{aligned}$$

2. Read the following and answer any four questions from (i) to (v).

The Great Stupa at Sanchi is one of the oldest stone structure in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BC. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. (Take $\pi = \frac{22}{7}$)

[CBSE Question Bank]

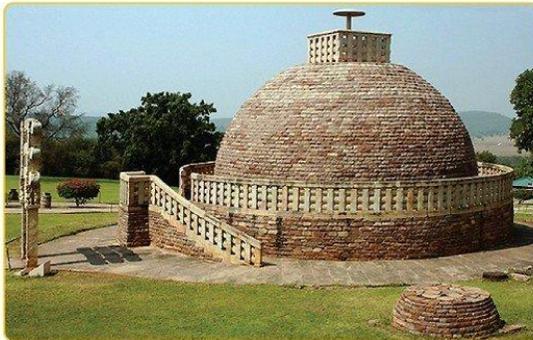
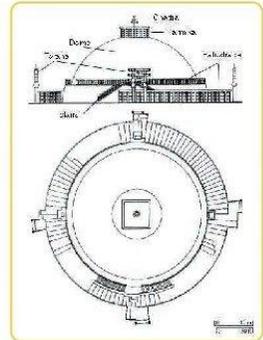


Fig. 12.29



(i) The volume of the hemispherical dome if the height of the dome is 21 m is

- (a) 19404 cu. m (b) 20000 cu. m (c) 15000 cu. m (d) 19000 cu. m

(ii) The formula to find the volume of sphere is

- (a) $\frac{2}{3} \pi r^3$ (b) $\frac{4}{3} \pi r^3$ (c) $4\pi r^2$ (d) $2\pi r^2$

(iii) The cloth required to cover the hemispherical dome if the radius of its base is 14 m, is

- (a) 1222 sq.m (b) 1232 sq.m (c) 1200 sq.m (d) 1400 sq.m

(iv) The total surface area of the combined figure i.e. hemispherical dome with radius 14m and cuboidal shaped top with dimensions 8 m × 6 m × 4 m is

- (a) 1200 sq. m (b) 1232 sq. m
(c) 1392 sq.m (d) 1932 sq. m

(v) The volume of the cuboidal shaped top with dimensions mentioned in part (iv) above, is

- (a) 182.45 m³ (b) 282.45 m³
(c) 292m³ (d) 192 m³

Sol. (i) Height of the dome = Radius of the dome = 21 m

i.e., $h = r = 21$ m

$$\begin{aligned}\text{Volume of the hemispherical dome} &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \\ &= 19404 \text{ cu. m}\end{aligned}$$

∴ Option (a) is correct.

(ii) Volume of sphere = $\frac{4}{3}\pi r^3$

∴ Option (b) is correct.

(iii) The cloth required to cover the hemispherical dome

$$\begin{aligned}&= \text{Curved surface area of dome} \\ &= 2\pi r^2 \\ &= 2 \times \frac{22}{7} \times 14 \times 14 \\ &= 1232 \text{ sq. m}\end{aligned}$$

∴ Option (b) is correct.

(iv) Total surface area of the combined dome with cuboidal shaped top

$$\begin{aligned}&= \text{CSA of dome} + 2(lb + bh + lh) - l \times b \\ &= 1232 + 2(48 + 24 + 32) - 48 \\ &= 1392 \text{ sq. m}\end{aligned}$$

∴ Option (c) is correct.

(v) Volume of the cuboidal shaped top = $l \times b \times h$

$$\begin{aligned}&= 8 \times 6 \times 4 \\ &= 192 \text{ m}^3\end{aligned}$$

∴ Option (d) is correct.

PROFICIENCY EXERCISE

■ Objective Type Questions:

[1 mark each]

1. Choose and write the correct option in each of the following questions.

- (i) The radius of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is
(a) 2.1 cm (b) 4.2 cm (c) 3.1 cm (d) 2.2 cm

- (ii) Manish wants to polish the object, which is composed of a cylinder surmounted by a hemisphere. If the whole length of the solid is 5 m and the diameter of the hemisphere is 7 m, what is the cost of polishing the surface area of the solid at the rate of 50 paise per sq. m.? (Use $\pi = \frac{22}{7}$)

[Competency Based Question]

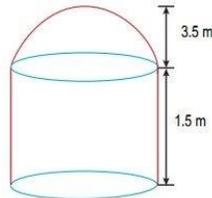


Fig. 12.30

- (a) ₹ 55 (b) ₹ 110 (c) ₹ 550 (d) ₹ 1,100
- (iii) A cylinder and a cone are of the same base radius and same height. The ratio of the volumes of the cylinder to that of the cone is
- (a) 1 : 3 (b) 1 : 2 (c) 3 : 1 (d) 2 : 1
- (iv) The length of the diagonal of a cube is $8\sqrt{3}$ cm. Its total surface area is
- (a) 321 cm^2 (b) 350 cm^2 (c) 384 cm^2 (d) 256 cm^2
- (v) The shape of a *gilli*, in the *gilli-danda* game figure is a combination of [NCERT Exemplar]



Fig. 12.31

- (a) two cylinders (b) a cone and a cylinder
- (c) two cones and a cylinder (d) two cylinders and a cone
- (vi) If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5 cm and height 6 cm, then how much water flows out of the cylindrical cup? [NCERT Exemplar]
- (a) 38.8 cm^3 (b) 55.4 cm^3 (c) 19.4 cm^3 (d) 471.4 cm^3

■ Very Short Answer Questions:

[1 mark each]

- Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.
- A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the ratio between radius of their bases to their height is 3:4. [CBSE 2019 (C) (30/1/1)]
- A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 2\pi r^2$. State true or false and justify. [NCERT Exemplar]
- The surface area of a sphere is 616 cm^2 . Find its radius. [NCERT Exemplar]
- The radii of two cylinders are in the ratio 2 : 5 and their heights are in the ratio 3 : 4. What is the ratio of their curved surface areas?

■ Short Answer Questions-I:

[2 marks each]

- A hemisphere and a cone have equal bases. If their heights are also equal, then what is the ratio of their curved surfaces?
- A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled? [CBSE 2019(30/4/2)]

- Two identical cubes of volume 343 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?
- Two cones with same base diameter 14 cm and height 24 cm are joined together along their bases. Find the surface area of the shape so formed.
- A cone of maximum size is carved out from a cube of edge 20 cm. Find the surface area of the solid left out after the cone is carved out.

■ Short Answer Questions-II:

[3 marks each]

- Find the number of shots, each having diameter 6 cm that can be made from a cuboidal lead which is solid of dimensions $24 \text{ cm} \times 22 \text{ cm} \times 12 \text{ cm}$.
- Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker, so that the water level rises by 5.6 cm.
- A wall 24 m long, 0.4 m thick and 6 m high which is constructed with the bricks, each of dimensions $25 \text{ cm} \times 16 \text{ cm} \times 10 \text{ cm}$. If the mortar occupies $\frac{1}{10}$ th of the volume of the wall, then find the number of bricks used in constructing the wall. [Competency Based Question]
- Two solid cones A and B are placed in a cylindrical tube in Fig. 12.32. The ratio of their capacities are $2 : 1$. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.

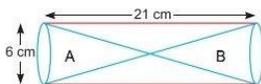


Fig. 12.32

- An ice-cream cone is full of ice-cream having radius 5 cm and height 10 cm (in Fig. 12.33). Calculate the volume of ice-cream, provided that its $\frac{1}{6}$ part is left unfilled with ice-cream.

[NCERT Exemplar]

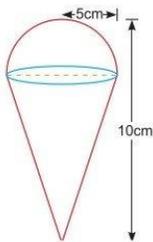


Fig. 12.33

- A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much height will the level of water rise in the cylindrical vessel? [CBSE (F) 2017]
- A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty the tank, if it is 3 m in diameter? (Take $\pi = \frac{22}{7}$) [CBSE 2019 (C) (30/1/1)]
- Two cones with same base diameter 16 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed. [CBSE 2019 (C) (30/1/1)]

■ Long Answer Questions:

[5 marks each]

- A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment. [CBSE Delhi 2015]
- A factory manufactures 18000 cylindrical boxes daily. The length of each box is 32 cm and circumference of base is 6 cm. Find the cost of colouring the curved surfaces of the boxes manufactured in one day at ₹ 0.08 per dm^2 .
- How many cubic centimeter of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm? If one cubic cm of iron weighs 7.5 g, find the weight of the box. [NCERT Exemplar]
- A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly all around it to a width of 5 m to form an embankment. Find the height of the embankment. [CBSE (F) 2017]
- A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy, if the total height of the toy is 30 cm.
- The interior of a building is in the form of a cylinder of diameter 4 m and height 3.5 m, surmounted by a cone of the same base with vertical angle as a right angle. Find the curved surface area and volume of the interior of the building.
- A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 9 cm and 15 cm, respectively. If the slant height of the conical portion is 7.5 cm, find the total surface area and volume of the rocket. [Competency Based Question]
- A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $54\frac{10}{21} \text{ m}^3$ of air. If the internal diameter of dome is equal to $\frac{4}{5}$ of the total height above the floor, find the height of the building. [Competency Based Question]
- Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm^3/sec in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?
- From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take $\pi = \frac{22}{7}$] [CBSE Delhi 2014]

Answers

- | | | | | | |
|---|---|--|--------------|-------------------|----------|
| 1. (i) (a) | (ii) (a) | (iii) (c) | (iv) (c) | (v) (c) | (vi) (a) |
| 2. $\frac{1}{9}$ | 4. True | 5. 7 cm | 6. 3 : 10 | 7. $\sqrt{2} : 1$ | |
| 8. 100 minutes or 1 hour 40 minutes | 9. 490 cm^2 | 10. 1100 cm^2 | | | |
| 11. $2400 - \frac{2200}{7}(1 - \sqrt{5}) \text{ cm}^2$ | 12. 56 | 13. 150 | 14. 12960 | | |
| 15. 14 cm, 7 cm, 132 cm^3 , 66 cm^3 , 396 cm^3 | 16. 327.4 cm^3 | 17. 1 cm | | | |
| 18. 1980 seconds or 33 minutes | 19. 854.08 cm^2 | | | | |
| 20. 10 m | 21. ₹ 2764.80 | 22. 3960 cm^3 , 29.7 kg | 23. 0.7875 m | | |
| 24. 770 cm^2 | 25. $(14 + 4\sqrt{2})\pi \text{ m}^2$, $\frac{50\pi}{3} \text{ m}^3$ | 26. 594 cm^2 , 1081.9 cm^3 | 27. 5 m | | |
| 28. 90 cm | 29. 73.92 cm^2 | | | | |

Self-Assessment

Time allowed: 1 hour

Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions. (3 × 1 = 3)
- (i) The ratio of volume of a cone and a cylinder of equal diameter and equal height is
(a) 3 : 1 (b) 1 : 3
(c) 1 : 2 (d) 2 : 1
- (ii) A cylinder, a cone and a hemisphere are on the same base and having same height. The ratio of their volumes is
(a) 1 : 2 : 3 (b) 2 : 1 : 3
(c) 3 : 1 : 2 (d) 3 : 2 : 1
- (iii) The radii of bases of cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3, then ratio between the volumes of cylinder and that of cone is
(a) 8 : 9 (b) 9 : 8
(c) 5 : 7 (d) 7 : 5
2. Solve the following questions. (2 × 1 = 2)
- (i) What geometrical shapes is the plumb line (Fig. 12.34) a combination of? [NCERT Exemplar]



Fig. 12.34

- (ii) What is the curved surface area of a right circular cone of height 24 cm and base diameter 14 cm?

SECTION B

- Solve the following questions. (4 × 2 = 8)
3. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?
4. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. If the cost of painting 1 cm^2 of the surface area is ₹ 0.05, find the total cost of painting the vessel all over. (Take $\pi = \frac{22}{7}$)
5. The rain water from a roof of dimensions 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall (in cm).
6. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 100 cm and the diameter of the hemispherical ends is 28 cm. Find the cost of polishing the surface of the solid at the rate of 5 paise per sq. cm.

■ Solve the following questions.

(4 × 3 = 12)

- A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice cream. The whole ice-cream is to be divided among 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone. [CBSE (F) 2016]
- The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one-fifth of a litre? [NCERT Exemplar]
- A cylinder, whose height is two-thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.
- Five hundred persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?

■ Solve the following questions.

(3 × 5 = 15)

- Water is flowing at the rate of 15km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm? [NCERT Exemplar]
- In Fig. 12.35, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. [Use $\pi = \frac{22}{7}$] [CBSE Delhi 2015]

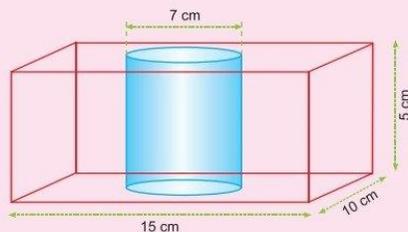


Fig. 12.35

- A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into the vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball. [CBSE (F) 2015]

Answers

- | | | | | |
|------------------------|------------------------------|-----------|----------------------------|-------------------------|
| 1. (i) (b) | (ii) (c) | (iii) (b) | 2. (i) Hemisphere and cone | (ii) 550 cm^2 |
| 3. $6 : \pi$ | 4. 1925.79 cm^2 | | 5. 2.5 cm | 6. ₹ 440 |
| 8. 480000 words | | | 7. 6 cm | |
| 12. 583 cm^2 | 13. $\frac{1}{2} \text{ cm}$ | | 9. 4 cm | 10. 0.5 cm |
| | | | 11. 2 hours | |

